A New Type of Location Problem

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Abstract If a company wants to found a supermarket in a city, no matter there have been supermarkets or not, it will definitely maximize its profit with some constraints as a business strategy. When the restriction is distance, the problem is abstracted as the maximing location problem with distance restriction (MLPWDR). An optimal algorithm for MLP – WDR is presented in this paper, and its complexity is analyzed, too.

Key words: location problem; optimal algorithm; complexity analysis; supermarket; business strategy
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一种新型的选址问题

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摘要: 某一公司想在某一城市投资一个超市, 无论这一城市是否已有一些超市, 作为一种商业行为, 该公司总是要使得自己的收益在某些条件的限制下达到最大, 当限制条件为距离时, 我们把这一问题抽象为具有距离限制的最大 loading选址问题(MLPWDR). 本文给出了该类问题的一个最优算法及其复杂性分析.

关键词: 选址问题; 最优算法; 复杂性分析; 超市; 经营策略

1 Introduction

Consider the following background: Given a set of regions of one city, if there is a path from region i to region j without pass through other region, then there is a distance d_{ij} and a maximium number of people who can go through at the same time capacity c_{ij} are specified, there is a number of people who live in region i (potential profit function) w_i for every region i, and a constant number d_c is given, too. Suppose some or all integer part of w_i are obtained by the supermarket through the path p from region i to the region j the supermarket located, such that $length(p)\ \sum_{ij\in p} d_{ij} \le d_c$, and for every pair ij, we use $f(ij)$ denotes all the number of people who pass through ij, $f(ij) \le c(ij)$. Now, there is no supermarket belonging to the company who wants to invest in the city, and there is a company who wants to found a supermarket in one of the regions such that its loading (profit) is maximized with above restrictions on distance and capacity. We name this problem as the Maximing loading location problem with distance restriction, denoted by MLPWDR.

Now, we treat every region i to vertex v_i, suppose there are n regions in this city, $i = 1, 2, \cdots, n$. If there is

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a path from region i to region j without pass through other region, then there is an edge v_iv_j, and set $d(v_iv_j) = d_{ij}$, $c(v_iv_j) = c_{ij}$, $w(v_i) = w(i)$, a constant number d_c is given, too. Then we can transform the MLPWDR problem to a graph theory problem as described in the following model.

Definition 1 (MLPWDR) Given an undirected and connected graph $G = (v, E; w, c, d)$ with a potential profit function $w: V \rightarrow Z^+$, a capacity function $c: E \rightarrow Z^+$, a distance (or cost) function $d: E \rightarrow R^+$, and a constant positive number d_c, $|V| = n$, $|E| = m$. The MLPWDR problem is how to find a vertex set $\{v_{location}\}$ which is a subset of V, such that the people flow from all possible vertices of V to $v_{location}$ is maximized, i.e. $loading(v_{location}) = maximize\{loading(v_i) \mid i = 1, 2, \cdots, n\}$.

Here, if there is people flow from v_j to $v_{location}$, then all the flow are flowed along the paths $p \in P$ with $length(p) \leq d_c$, and for every edge v_iv_j, the flow value $f(v_iv_j)$ passing through it satisfies $f(v_iv_j) \leq c(v_iv_j)$.

There are a lot of results about location problem, such as the $k-Center$ problem and the $k-Median$ problem[1]. But these two location problems are different from our location problem, because these models are different from each other. The first model is given a set of cities, with intercity distances specified, the objective is how to pick k cities for locating warehouses so as to minimize the maximum distance of a city from its closest warehouse; this problem is called $k-Center$ problem. And the second model is that G is a bipartite graph with bipartition (F, C), where F is the set of facilities and C is the set of cities, and k is a positive integer to specify the number of facilities and that are allowed to be opened. Let c_{ij} be the cost to connect city j to (opened) facility i. The $k-Median$ problem is how to find a subset $I \subseteq F$, $|I| \leq k$, of facilities that will be opened and a function $\alpha: C \rightarrow I$ is how to assign cities to opened facilities in such a way that the total connecting cost is minimized. References [2] and [3] considered a variant of $k-Center$ model and a variant of $k-Median$ problem, respectively, they are not the same model as here. Inge Li Gortz and Anthony Wirth[4] studied asymmetry weighted $k-Center$ problem, and they gave an $O(\log n)$ - approximate algorithm for the problem. The model here is different from the preceding models in the distance restriction.

Because the MLPWDR problem is very crisis for an investor, we study it in this paper, and then we give an optimal algorithm for the MLPWDR problem by traversing methods, and we finally present analysis of the algorithm.

We present an optimal algorithm for MLPWDR and analyze its complexity in Section 2, and conclude our paper with some discussions in Section 3.

2 An Optimal Algorithm for the MLPWDR Problem

Here, we study the MLPWDR problem. Combining traversing method and Max- flow algorithm, we design an optimal algorithm for the MLPWDR problem. The algorithm Dijkstra how to find max- flow by augmenting shortest path algorithm on directed networks is used in our algorithm.

Now, we can design the algorithm.

Algorithm 1 MLPWDR.

INPUT: an undirected and connected graph $G = (V, E; w, c, d)$ and a constant positive number d_c.

OUTPUT: an optimal solution of the MLPWDR problem $\{v_{loacation}\}$ such that $loading(v_{location} = maximize\{loading(v_i) \mid i = 1, 2, \cdots, n\}$.

Begin

set $i = 1$

While $(i \neq n + 1)$, do

Step 1 Find the location on vertex v_i, $loading(v_i) \Leftarrow w(v_i)$, and set the initially flow $f = 0$.

Step 2 In this step, we construct an auxiliary directed three partite network $N = (\bar{V}, A, \bar{c}, \bar{d})$, $\bar{V} = \{s\} \cup V$,

s is an added origin vertex, v_i is a sink vertex $A = A(s, v_j) \cup A(E), A(s, v_j)$ is an arc set from s to v_i for every $v_j \in V - \{v_i\}$, and for every edge $e = v_j v_k \in E$, if $v_i \in \{v_j, v_k\}$, then the direction of the arc $A(e)$ is only to v_i, else the directions of the arc $A(e)$ are to both v_j and v_k. set $\bar{c}(sv_j) = w(v_j)$, $\bar{c}(A(v_j v_k)) = c(E(v_j v_k))$, and $\bar{d}(sv_j) = 0$, $\bar{d}(A(v_j v_k)) = d(E(v_j v_k))$.

Step 3 Find a shortest augmenting path $\bar{p}$ of this flow f from s to v_i by the algorithm Dijkstra. If $length(\bar{p}) = \sum_{arc(v_j v_k) \in \bar{p}} \bar{d}(A(v_j v_k)) d_c$, then stop; Else, by augmenting the path $\bar{p}$ by the algorithm Dijkstra, suppose the augmenting flow value is $aug(f)$, set $f = f + aug(f)$, $loading(v_i) = loading(v_i) + aug(f)$.

Step 4 Output $loading(v_i)$, $i \Leftarrow i + 1$.

Now, we analyze the algorithm of its optimality by the following theorem.

Theorem 1 Algorithm 1 is an optimal algorithm in $O(n^3)$ for the MLPWDR problem.

Proof To prove this theorem, we suppose the Output $\{v_{location}\}$ of Algorithm 1 is not an optimal solution for the MLPWDR problem. Then we can suppose there is an optimal solution v_{l1} for the MLPWDR problem with maximizing *loading*. The *loadings* denoted by $loading(v_{loacation})$ and $loading(v_{l1})$, respectively, then $loading(v_{loacation}) \leq loacing(v_{l1})$ is satisfied.

And when we consider $i = l1$ in our algorithm, the max-flow from s to v_{l1} under the restrictions of capacity and distance d_c can be computed optimally by the algorithm Dijkstra, but $v_{l1} \notin \{v_{location}\}$, that is to say the inequality $loading(v_{loacation}) > loading(v_{l1})$ is satisfied when Algorithm 1 stops.

It is easily to notice the contradiction between the above two inequalities, then our assumption that the Output $\{v_{location}\}$ of Algorithm 1 is not an optimal solution of the MLPWDR problem is incorrect.

Now, we analyze the complexity of every step in Algorithm 1. The circulation “while” can be executed in $O(n)$ unit time, the complexity of executing Step 1 and Step 2 is $O(n+m)$, the algorithm Dijkstra in Step 3 can be executed in time $O(n^2)$, so the complexity of Step 3 is $O(n^2)$, and Step 4 can be executed in $O(n)$. Therefore the proof is completed.

3 Conclusion

In this paper, we study a new model of location problem named as the MLPWDR problem, then we design an optimal algorithm for the MLPWDR problem by traversing method and analyze the complexity of the algorithm. But we should remark that when there have been $(k-1)$ different supermarkets $s_2, s_2, \cdots, s_{k-1}$ occupied $(k-1)$ different regions in the city, this problem is called the kth Maximizing loading location problem with distance restriction, our algorithm is efficient, too. We only need to verify the algorithm as the following method.

If there are $(k-1)$ supermarkets which have been founded in the city, then we set the regions occupied as $1, 2, \cdots, (k-1)$, and the related vertices are $v_1, v_2, \cdots, v_{k-1}$. And in our algorithm we set $i = k$ for Begin step simply, the other steps invariantly.

References

[1] VAZIRANI V V. Approximation Algorithms[M]. Hong Kong: Springer, 2001.

[2] Guang Xu, Jinhui Xu. An Improved Approximation Algorithm for Uncapacited Facility Location Problem with Penalties[J]. Lecture Notes in Computer Science, Springer-Verlag Berlin Heidelberg, 2005, 3595: 644-653.

[3] Marek Chrobak, Claire Kenyon, Neal E. Yong. The Reverse Greedy Algorithm for the Metric K-Median Problem[J]. Lecture Notes in Computer Science, Springer-Verlag Berlin Heidelberg, 2005, 3595: 654-660.

[4] Inge Li Gortz, Anthony Wirth. Asymmetry in Variants[M]. Preprint submitted to Elsevier, 2005.