具有偏差变元的Volterra型系统的周期正解 ①

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摘要　利用Mawhin重合度理论研究了一类具有偏差变元的Volterra型系统周期正解的存在性问题, 得到了一个新的存在性定理.

关键词: Volterra型系统; 周期正解; 存在性; 重合度

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0 引 言

考虑下列三个种群相互作用的Volterra型系统

$$\begin{cases} \dot N_1(t) = N_1(t)[r_1(t) - a_{11}(t)N_1(t-\tau_{11}(t)) - a_{12}(t)N_2(t-\tau_{12}(t)) - a_{13}(t)N_3(t-\tau_{13}(t))] \\ \dot N_2(t) = N_2(t)[-r_2(t) + a_{21}(t)N_1(t-\tau_{21}(t)) - a_{23}(t)N_3(t-\tau_{23}(t))] \\ \dot N_3(t) = N_3(t)[-r_3(t) + a_{31}(t)N_1(t-\tau_{31}(t)) - a_{32}(t)N_2(t-\tau_{32}(t))] \end{cases} \quad (1)$$

其中 $r_i(t), a_{ij}(t), \tau_{ij}(t) \quad (i, j = 1, 2, 3): R \to [0, +\infty]$ 都是连续的 ω 周期函数, $a_{22}(t) = a_{33}(t) = 0$.

由系统(1)所代表的生态学意义, 我们仅关心其正解. 为此, 考虑(1)的如下初值条件

$$N_i(s) = \Phi_i(s) \in C([-r, 0], [0, \infty)), \ \Phi_i(0) > 0, (i = 1, 2, 3) \quad (2)$$

其中 $r = \max_{1 \le i,j \le 3}\{\max_{t \in [0,\omega]} \tau_{ij}(t)\}$.

由于Volterra型系统在生态学中具有十分重要的意义, 因此近年来倍受生物数学界关注. 但以往的文献大都局限于对其解的稳定性, 吸引性, 持久性, 振动性等问题的研究[1~4], 关于初值问题(1)和(2)的周期正解的全局存在性的工作则还不多. 本文的目的在于建立系统(1)的周期正解的全局存在性定理, 给出一个新的简洁实用的判别准则. 下面先简单介绍一下本文将要用到的重合度理论的有关知识.

设 X, Z 是两个Banach空间. 考虑算子方程 $Lx = \lambda Nx, \lambda \in [0, 1]$, 这里 $L: \mathrm{Dom}L \cap X \to Z$ 是线性算子. 定义两个投影算子分别为 $P: X \cap \mathrm{Dom}L \to X, Q: Z \to Z/\mathrm{Im}L$ 使得 $\mathrm{Im}P = KerL, \mathrm{Im}L = \mathrm{Ker}Q$. 后面我们将要用到Mawhin[5]的如下结果:

引理　设 x, z 是Banach空间, L 是指标为0的Fredholm算子, 在 $N: \bar\Omega \to Z$ 在 $\bar\Omega$ 是 L 紧的, 其中 Ω 是 X 中的有界开集, 且满足

(i) $Lx \neq \lambda Nx, \forall x \in \partial\Omega \cap \mathrm{Dom}L, \lambda \in (0, 1)$

(ii) $QNx \neq 0, \forall x \in \partial\Omega \cap \mathrm{Ker}L$

(iii) $\deg\{QNx, \Omega \cap \mathrm{Ker}L, 0\} \neq 0$

则方程 $Lx = Nx$ 在 $\bar\Omega$ 中至少存在一个解.

2 主要结果

定理　如果下列条件成立:

(a) $a_{11}^{(m)} a_{23}^{(m)} a_{32}^{(m)} + a_{12}^{(m)} a_{23}^{(m)} a_{31}^{(m)} + a_{13}^{(m)} a_{21}^{(m)} a_{32}^{(m)} > 0$

$a_{13}^{(m)} a_{32}^{(m)} \bar r_2 + a_{12}^{(m)} a_{23}^{(m)} \bar r_3 + a_{23}^{(m)} a_{32}^{(m)} \bar r_1 > 0$

(b) $a_{11}^{(m)} a_{23}^{(m)} \bar r_3 + a_{13}^{(m)} + a_{21}^{(m)} \bar r_3 - a_{13}^{(m)} a_{31}^{(m)} \bar r_2 - a_{23}^{(m)} a_{31}^{(m)} \bar r_1 < 0$

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$$a_{11}^{(M)}a_{32}^{(M)}\bar{r}_2+a_{12}^{(M)}a_{31}^{(M)}\bar{r}_2-a_{21}^{(m)}a_{32}^{(m)}\bar{r}_1-a_{12}^{(m)}a_{21}^{(m)}\bar{r}_3<0$$

其中

$$\bar{r}_i=\frac{1}{\omega}\int_0^\omega r_i(t)\,dt,\ a_{ij}^{(M)}=\max_{i\in[0,\omega]}\frac{a_{ij}(t)}{1-\tau'_{ij}(t)},\ a_{ij}^{(m)}=\min_{i\in[0,\omega]}\frac{a_{ij}(t)}{1-\tau'_{ij}(t)}$$

$$\tau'_{ij}(t)<1,(i,j=1,2,3)$$

则初值问题(1),(2)至少存在一个 ω 周期正解.

证　因为问题(1),(2)的解对 $t\geqslant 0$ 有 $N_t(t)>0(i=1,2,3)$. 故可作变换

$$N_i(t)=e^{x_i(t)},(i=1,2,3) \tag{3}$$

在变换(3)下,(1)化为

$$\begin{cases}\dot{x}_1(t)=r_1(t)-a_{11}(t)e^{x_1(t-\tau_{11}(t))}-a_{12}(t)e^{x_2(t-\tau_{12}(t))}-a_{13}(t)e^{x_3(t-\tau_{13}(t))}\\ \dot{x}_2(t)=-r_2(t)+a_{21}(t)e^{x_1(t-\tau_{21}(t))}-a_{23}(t)e^{x_3(t-\tau_{23}(t))}\\ \dot{x}_3(t)=-r_3(t)+a_{31}(t)e^{x_1(t-\tau_{31}(t))}-a_{32}(t)e^{x_2(t-\tau_{32}(t))}\end{cases} \tag{4}$$

取 $X=Z=\{x(t)=(x_1(t),x_2(t),x_3(t)(t))^T\in C(R,R^3):x(t+\omega)=x(t)$,

记 $\|x(t)\|=\max_{t\in[0,\omega]}|x_1(t)|+\max_{t\in[0,\omega]}|x_2(t)|+\max_{t\in[0,\omega]}x_3(t)|$, 则 X,Z 在模 $\|\cdot\|$ 下成为 Banach 空间.

令

$$Lx=\frac{dx}{dt}$$

$$Nx=\begin{bmatrix}r_1(t)-a_{11}(t)e^{x_1(t-\tau_{11}(t))}-a_{12}(t)e^{x_2(t-\tau_{12}(t))}-a_{13}(t)e^{x_3(t-\tau_{13}(t))}\\ -r_2(t)+a_{21}(t)e^{x_1(t-\tau_{21}(t))}-a_{23}(t)e^{x_3(t-\tau_{23}(t))}\\ -r_3(t)+a_{31}(t)e^{x_1(t-\tau_{31}(t))}-a_{32}(t)e^{x_2(t-\tau_{32}(t))}\end{bmatrix}$$

$$Px=\frac{1}{\omega}\int_0^\omega x(t)\,dt,\ x\in X$$

$$Qz=\frac{1}{\omega}\int_0^\omega z(t)\,dt,\ z\in Z$$

易知,L 是指标为 0 的 Fredholm 算子,对 X 中的任一有界开集 Ω,N 在 $\bar{\Omega}$ 上是 L 紧的.

对应于算子方程 $Lx=\lambda Nx$, $\lambda\in(0,1)$, 有

$$\begin{cases}\dot{x}_1(t)=\lambda[r_1(t)-a_{11}(t)e^{x_1(t-\tau_{11}(t))}-a_{12}(t)e^{x_2(t-\tau_{12}(t))}-a_{13}(t)e^{x_3(t-\tau_{13}(t))}]\\ \dot{x}_2(t)=\lambda[-r_2(t)+a_{21}(t)e^{x_1(t-\tau_{21}(t))}-a_{23}(t)e^{x_3(t-\tau_{23}(t))}]\\ \dot{x}_3(t)=\lambda[-r_3(t)+a_{31}(t)e^{x_1(t-\tau_{31}(t))}-a_{32}(t)e^{x_2(t-\tau_{32}(t))}]\end{cases} \tag{5}$$

设 $x(t)\in X$ 是(5)对应于某一 $\lambda\in(0,1)$ 的解, 上式两端同时从 0 到 ω 积分得

$$\int_0^\omega a_{11}(t)e^{x_1(t-\tau_{11}(t))}dt+\int_0^\omega a_{12}(t)e^{x_2(t-\tau_{12}(t))}dt+\int_0^\omega a_{13}(t)e^{x_3(t-\tau_{13}(t))}dt=\omega\bar{r}_1 \tag{6}$$

$$\int_0^\omega a_{21}(t)e^{x_1(t-\tau_{21}(t))}dt-\int_0^\omega a_{23}(t)e^{x_3(t-\tau_{23}(t))}dt=\omega\bar{r}_2 \tag{7}$$

$$\int_0^\omega a_{31}(t)e^{x_1(t-\tau_{31}(t))}dt-\int_0^\omega a_{32}(t)e^{x_2(t-\tau_{32}(t))}dt=\omega\bar{r}_3 \tag{8}$$

令 $s=t-\tau_{ij}(t)$, 因为 $\tau'_{ij}<1$,, 故 $\frac{ds}{dt}=1-\tau'_{ij}(t)>0$, $ds=(1-\tau'_{ij}(t))dt$, 由此即知 s 关于 t 严格单增, 于是 $s=t-\tau_{ij}(t)$ 必存在连续的反函数 $t=\tau_{ij}^*(s)$, 其中 $s\in[-\tau_{ij}(0),\omega-\tau_{ij}(0)]$, 从而有

$$\int_0^\omega a_{ij}(t)e^{x_{ij}(t-\tau_{ij}(t))}dt=\int_{-\tau(0)}^{\omega-\tau_{ij}(\omega)}e^{x_{ij}(s)}\frac{a_{ij}(\tau_{ij}^*(s))}{1-\tau'_{ij}(\tau_{ij}^*(s))}ds.$$

由广义积分中值定理可知, $\exists\,\xi_{ij}\in[-\tau_{ij}(0),\omega-\tau_{ij}(\omega)]=[-\tau_{ij}(0),\omega-\tau_{ij}(0)]$ 使得

$$\int_0^\omega a_{ij}(t)\,e^{x_{ij}(t-\tau_{ij}(t))}dt=\frac{a_{ij}(\tau_{ij}^*(\xi_{ij}))}{1-\tau'_{ij}(\tau_{ij}^*(\xi_{ij}))}\int_{-\tau_{ij}(0)}^{\omega-\tau_{ij}(\omega)}e^{x_j(s)}ds=\frac{a_{ij}(\eta_{ij})}{1-\tau'_{ij}(\eta_{ij})}\int_0^\omega e^{x_j(s)}ds.$$

(其中 $\eta_{ij}=\tau^*(\xi_{ij})$).

记 $b_{ij}=\dfrac{a_{ij}(\eta_{ij})}{1-\tau'_{ij}(\eta_{ij})}$, 从而(6), (7), (8) 变为

$$b_{11}\int_0^\omega e^{x_1(s)}ds+b_{12}\int_0^\omega e^{x_2(s)}ds+b_{13}\int_0^\omega e^{x_3(s)}ds=\omega\bar r_1 \tag{9}$$

$$b_{21}\int_0^\omega e^{x_1(s)}ds-b_{23}\int_0^\omega e^{x_3(s)}=\omega\bar r_2 \tag{11}$$

$$b_{31}\int_0^\omega e^{x_1(s)}ds-b_{32}\int_0^\omega e^{x_2(s)}ds=\omega\bar r_3 \tag{11}$$

显然在定理的条件下必有 $\int_0^\omega e^{x_j(s)}ds=\dfrac{D_j}{D}$, 其中

$$D=\begin{vmatrix} b_{11} & b_{12} & b_{13}\\ b_{21} & 0 & -b_{23}\\ b_{31} & -b_{32} & 0\end{vmatrix}=-(b_{11}b_{23}b_{32}+b_{12}b_{23}b_{31}+b_{13}b_{21}b_{32})<0,$$

$$D_1=\begin{vmatrix} \omega\bar r_2 & b_{12} & b_{13}\\ \omega\bar r_2 & 0 & -b_{23}\\ \omega\bar r_3 & -b_{32} & 0\end{vmatrix}=-\omega(b_{12}b_{23}\bar r_3+b_{13}b_{32}\bar r_2+b_{23}b_{32}\bar r_1<0,$$

$$D_2=\begin{vmatrix} b_{11} & \omega\bar r_1 & b_{13}\\ b_{21} & \omega\bar r_2 & -b_{23}\\ b_{31} & \omega\bar r_3 & 0\end{vmatrix}=\omega(b_{11}b_{23}\bar r_3+b_{13}b_{21}\bar r_3-b_{13}b_{31}\bar r_2-b_{23}b_{31}\bar r_1<0,$$

$$D_3=\begin{vmatrix} b_{11} & b_{12} & \omega\bar r_1\\ b_{21} & 0 & \omega\bar r_2\\ b_{31} & -b_{32} & \omega\bar r_3\end{vmatrix}=\omega(b_{11}b_{32}\bar r_2+b_{12}b_{31}\bar r_2-b_{12}b_{21}\bar r_3-b_{21}b_{32}\bar r_1<0.$$

从而 $\exists\, t_j\in[0,\omega]\,(j=1,2,3)$ 使得

$$e^{x_j(t_j)}=\frac{D_j}{D},\ 或\ x_j(t_j)=\ln\frac{D_j}{D},$$

显然

$$\frac{D_1}{D}\leqslant\frac{a_{12}^{(M)}a_{23}^{(M)}\bar r_3+a_{13}^{(M)}a_{32}^{(M)}\bar r_2+a_{23}^{(M)}a_{32}^{(M)}\bar r_1}{a_{11}^{(M)}a_{23}^{(M)}a_{32}^{(M)}+a_{12}^{(m)}a_{23}^{(m)}a_{31}^{(m)}+a_{13}^{(m)}a_{21}^{(m)}a_{32}^{(m)}},$$

$$\frac{D_1}{D}\geqslant\frac{a_{12}^{(m)}a_{23}^{(m)}\bar r_3+a_{13}^{(m)}a_{32}^{(m)}\bar r_2+a_{23}^{(m)}a_{32}^{(M)}\bar r_1}{a_{11}^{(M)}a_{23}^{(M)}a_{32}^{(M)}+a_{12}^{(M)}a_{23}^{(M)}a_{31}^{(M)}+a_{13}^{(M)}a_{21}^{(M)}a_{32}^{(M)}}.$$

同理可证 $\dfrac{D_2}{D}$, $\dfrac{D_3}{D}$ 有界, 从而 $x_j(t_j)=\ln\dfrac{D_j}{D}$ 有界, 故必存在常数 M_j 使得

$$|x_j(t_j)|\leqslant M_j\quad(j=1,2,3) \tag{12}$$

由(5), (6) 可知

$$\int_0^\omega|x_1(t)|dt=\lambda\int_0^\omega|r_1(t)-a_{11}(t)e^{x_1(t-\tau_{11}(t))}-a_{12}(t)e^{x_2(t-\tau_{12}(t))}-a_{13}(t)e^{x_3(t-\tau_{13}(t))}|dt<$$

$2\lambda\omega\bar r_1<2\omega\bar r_1$.

从而由(12), (13) 便知

$$|x_1(t)|\leqslant|x_1(t_1)|+\int_0^\omega|x_1(t)|dt\leqslant M_1+2\omega\bar r_1=:M_4.$$

此外, 由(5), (7) 便知

$$\int_0^\omega|x_2(t)|dt=\lambda\int_0^\omega|-r_2(t)+a_{21}e^{x_1(t-\tau_{21}(t))}-a_{23}(t)e^{x_3(t-\tau_{23}(t))}|dt$$

$$< \int_0^\omega r_2(t)\mathrm{d}t + \int_0^\omega a_{21}(t)\mathrm{e}^{x_1(t-\tau_{21}(t))}\mathrm{d}t + \int_0^\omega a_{23}(t)\mathrm{e}^{x_3(t-\tau_{23}(t))}\mathrm{d}t = 2\int_0^\omega a_{21}(t)\mathrm{e}^{x_1(t-\tau_{21}(t))}\mathrm{d}t$$

$$\leqslant 2\mathrm{e}^{M_4}\int_0^\omega a_{21}(t)\mathrm{d}t. \tag{14}$$

从而由(12), (14) 便知

$$|x_2(t)| \leqslant x_2(t_2)| + \int_0^\omega |x_2(t)|\mathrm{d}t \leqslant M_2 + 2\mathrm{e}^{M_4}\int_0^\omega a_{21}(t)\mathrm{d}t = : M_5.$$

同理由(5, (8) 及(12) 可证 $|x_3(t)| \leqslant M_6$, ($M_6 > 0$ 为一常数).

显然 M_j ($j = 1, 2, 3, 4, 5, 6$) 均与 λ 无关.

注意到 $\bar{a}_{ij} = \frac{1}{\omega}\int_0^\omega a_{ij}(t)\mathrm{d}t = \frac{1}{\omega}\int_{-\tau_{ij}(0)}^{\omega-\tau_{ij}(0)} \frac{a_{ij}(\tau_{ij}^*(s))}{1-\tau'_{ij}(s))}\mathrm{d}s = \frac{a_{ij}(\tau_{ij}^*(s_j))}{1-\tau'_{ij}(\tau_{ij}^*(s_j))}$.

因此, 在定理的条件下, 线性方程组

$$\begin{cases} \bar{a}_{11}u + \bar{a}_{12}v + \bar{a}_{13}w = \bar{r}_1, \\ \bar{a}_{21}u - \bar{a}_{23}w = \bar{r}_2, \\ \bar{a}_{31}u - \bar{a}_{32}v = \bar{r}_3. \end{cases}$$

有唯一正解(u^*, v^*, w^*).

令 M 充分大使得

$$M > M_4 + M_5 + M_6 + |\ln u^*| + |\ln v^*| + |\ln w^*|,$$

取 $\Omega = \{x(t) = (x_1(t), x_2(t), x_3(t))^T \in X: \|x\| < M\}$, 则对任一 $x \in \partial\Omega$, $\lambda \in (0, 1)$, 有 $Lx \neq \lambda Nx$. 又当 $x \in \partial\Omega \cap \mathrm{Ker}L = \partial\Omega \cap R^3$ 时, x 是 R^3 中的常向量且 $\|x\| = M$, 故

$$QNx = \begin{bmatrix} \bar{r}_1 - \bar{a}_{11}\mathrm{e}^{x_1} - \bar{a}_{12}\mathrm{e}^{x_2} - \bar{a}_{13}\mathrm{e}^{x_3} \\ -\bar{r}_2 + \bar{a}_{21}\mathrm{e}^{x_1} - \bar{a}_{23}\mathrm{e}^{x_3} \\ -\bar{r}_3 + \bar{a}_{31}\mathrm{e}^{x_1} - \bar{a}_{32}\mathrm{e}^{x_2} \end{bmatrix} \neq 0.$$

可直接计算得 $\deg\{QNx, \Omega \cap \mathrm{Ker}L, 0\} = 1$. 故在 $\bar{\Omega}$ 上引理的条件均成立, 从而由引理便知系统(4) 在 $\bar{\Omega}$ 中至少存在一个 ω 周期解, 再由变换(3) 即知系统(1) 存在 ω 周期正解. 证毕.

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Positive Periodic Solutions For Vol Terra- Type Systems with Deviating Arguments

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Abstract By using Mawhin coincidence degree, we study the global existence of positive periodic solutions for a class of Volterra- type systems with deviating arguments is studied and a new existence theorem is obtained.

Key words: Volterra- type system; positive periodic solution; existence; coincidence degree.