Influential Assessment in a Growth Curve Model with Rao's Simple Covariance Struture①

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Abstract Generalized influence function and generalized Cook distance are used to study local influence of observations on the parameter estimation in the growth cruve model with an unknown Rao's simple covariance structure. The diagnostics under the perturbations of error variance, response variables and design variables are derived. An example are analyzed for illustration.

Key words: growth curve model; rao's simple structure; generaliged influence function; generaliged Cook distance

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0 Introduction

Consider the growth curve model with Rao's simple structure[1]

$$Y_{p\times n} = X_{p\times q}B_{q\times k}Z_{k\times n} + E_{p\times n} \quad (1)$$

where B is a $q\times k$ unknown parameter matrix, Y is a $p\times n$ response matrix, X and Z are known design matrices of rank $q<p$ and $k<n$, we assume that column of matrix E are independent p~ variate normal distribution with mean vector zero and $p\times p$ covariance matrix $\Sigma>0$, i.e, $Y\sim N_{p\times n}(XBZ, I_n\otimes\Sigma)$, where $\otimes$ denotes the kronecker product, and

$$\Sigma = XX^T + Q\theta Q^T \quad (2)$$

where P and θ are $q\times q$, $(p-q)\times(p-q)$ unknown position definite matrices, and Q is a $p\times(p-q)$ matrix with rank $p-q$ such that $X^TQ=0$.

It is known that the maximun likelihood estimate(MLE) of unknown parameter B and Σ in this model are[2]:

$$\hat{B} = (X^TX)^{-1}X^TYZ^T(ZZ^T)^{-1} \quad (3)$$

$$\hat{\Sigma} = \frac{1}{n}[P_XSP_X + (I-P_X)YY^T(I-P_X)] \quad (4)$$

where, $S = Y[I-Z^T(ZZ^T)^{-1}Z]Y^T$, $P_X = X(X^TX)^{-1}X^T$

In this paper, we use the definitions of generalized influence function (GIF) and generalized Cook distance (GC)[3,4] to study local inflence in model(1).

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1 Local Influence of MLES of B

1.1 Perturbation of Error Variance

we first consider the situation that the perturbation is introduced to the variance of error variable.

$$\Sigma_W(E) = W^{-1} \otimes \Sigma \tag{5}$$

where, $W = \mathrm{diag}(w_1, \cdots, w_n)$, $w = (w_1, \cdots, w_n)^T$, The LSE of B under the perturbation(5) is $\hat{B}(W) = (X^TX)^{-1}X^TYWZ^T(ZWZ^T)^{-1}$. Let

$$h = (h_1, \cdots, h_n), w_0 = (1, \cdots 1)^T. \text{ Then we have:}$$

Theorem1: Under the perturbation (5), when $\hat{B}$ is of interest, the GIF and GC are given by:

$$GIF(\hat{B}H) = (X^TX)^{-1}X^TYQZ^THZ^T(ZZ^T)^{-1} \tag{6}$$

$$GC(\hat{B}, H) = | h^TD^{(1)}h| \tag{7}$$

where, $D^{(1)} = Q_{Z^T}Y^TX(X^T\hat{\Sigma}X)^{-1}X^TQ_{Z^T} \circ P_{Z^T}$

$$P_{Z^T} = Z^T(ZZ^T)^{-1}Z, \quad Q_{Z^T} = I - P_{Z^T}$$

$A \circ B$ denotes Hadarmand product[5]

Proof The Taylor series expansion of $\hat{B}(W)$ around $\varepsilon = 0$ is:

$$\hat{B}(W) = (X^TX)^{-1}X^TYWZ^T(ZWZ^T)^{-1}$$

$$= (X^TX)^{-1}X^TY(I + \varepsilon H)Z^T[Z(I + \varepsilon H)Z^T]^{-1}$$

$$\hat{B}(W) = \hat{B} + \varepsilon \hat{B}^{(1)}(h) + o(\varepsilon^2) \tag{8}$$

$$B^{(1)}(h) = (X^TX)^{-1}X^TY(I - P_{Z^T})HZ^T(ZZ^T)^{-1} = GIF(\hat{B}, H) \tag{9}$$

similar to case deletion, we use $M = ZZ^T$ and $C = (X^TX)^{-1}X^T\hat{\Sigma}X(X^TX)^{-1}$
This kind of choices for M and C hold through out in this paper. so we have:

$$GC(\hat{B}, H) = | tr\{B^{(1)}(h)MB^{(1)T}(h)C^{-1}\}|$$

$$= | tr\{H^TQ_{Z^T}Y^TX(X^T\hat{\Sigma}X)^{-1}X^TYQ_{Z^T}HP_{Z^T}\}|$$

$$| h^T\{Q_{Z^T}Y^TX(X^T\hat{\Sigma}X)^{-1}X^TYQ_{Z^T} \circ P_{Z^T})\}h|$$

Theorem 1 is proved.

Thus $h_{\max}(\hat{B})$ is the eigenvector associated with the largest absolute of $D^{(1)}$.

1.2 Perturbation of Response

Consider following perturbation scheme

$$Y_W = Y + KWL \tag{10}$$

where, $W = \varepsilon H$, $H = (h_{ij})_{p \times n}$, K, L denotes scale matrices accounting for the different measurement unit associated with Y. K is a $p \times p$ matrix. For example, if the kth column of Y is considesred. Let $K = I_{p \times p}$, $L = E_{k \times k}$, where $I_{p \times p}$ denotes identy martix of order p, $E_{k \times k}$ denotes a $n \times n$ matrix with the kth component is 1 and the other zero etc.

Theorem 2: Under the perturbation (10), if B is of interest, then the GIF and GC are:

$$GIF(\hat{B}, H) = (X^T X)^{-1} X^T K H L Z^T (ZZ^T)^{-1} \tag{11}$$

$$GC(\hat{B}, H) = vec^T(H) D^{(2)} vec(H)\} \tag{12}$$

where $D^{(2)} = L P_{Z^T} L^T \otimes K^T X (X^T \hat{\Sigma} X)^{-1} X^T K$

1.3 perturbation of Design Matrix X

Cosider following perturbation scheme

$$X_w = X + KWL \tag{13}$$

where $W = \varepsilon H$, $H = (h_{ij})_{p\times q}$, K, L denotes scale matrix.

Theorem 3: Under perturbation(13), GIF and GC are:

$$GIF(\hat{B}, H) = (X^T X)^{-1} X^T Y Q_{Z^T} L^T H^T K^T (ZZ^T)^{-1}$$

$$-(X^T X)^{-1} X^T Y Z^T (ZZ^T)^{-1} K H L Z Z^T (Z^T)^{-1} \tag{14}$$

$$GC(\hat{B}, H) = |\,\mathrm{vec}^T(H) D^{(3)} \mathrm{vec}(H)\} \tag{15}$$

where, $D^{(3)} = (E_2^T \otimes E_1) - U^T (E_4^T \otimes E_3) - (E_4 \otimes E_3^T) U + (E_6^T \otimes E_5)$

$$E_1 = K^T (ZZ^T)^{-1} K,\ E_2 = L Q_{Z^T} Y^T X (X^T \hat{\Sigma} X)^{-1} X^T Y Q_{Z^T} L^T$$

$$E_3 = L Q_{Z^T} Y^T X (X^T \hat{\Sigma} X)^{-1} X^T Y Z^T (ZZ^T)^{-1} K;\ \ E_4 = L Z^T (ZZ^T)^{-1} K$$

$$E_5 = K^T (ZZ^T)^{-1} Z Y^T X (X^T \hat{\Sigma} X)^{-1} X^T Y Z^T (ZZ^T)^{-1} K;\ E_6 = L P_{Z^T} L^T$$

1.4 perturbation of Design z

suppose perturbation scheme

$$Z_W = Z + KWH \tag{16}$$

where, $W = \varepsilon H$, $H = (h_{ij})_{k\times n}$, K, L denotes scale matrix.

Theorem 4: Under perturbation(16), GIF and GC are:

$$GIF(\hat{B}, H) = (X^T X)^{-1} L^T H^T K^T Q_X Y Z^T (ZZ^T)^{-1} - (X^T X)^{-1} X^T K H L (X^T X)^{-1} X^T Y Z^T (ZZ^T)^{-1} \tag{17}$$

$$GC(\hat{B}, H) = vec^T(H) D^{(4)} vec(H)\} \tag{18}$$

where, $D^{(4)} = (V_2^T \otimes V_1) - 2U^T (V_4^T \otimes V_3) + V_6^T \otimes V_5$

$$V_1 = K^T Q_X Y P_{Z^T} Y_Q^T X K,\quad V_2 = L (X^T \hat{\Sigma} X)^{-1} L$$

$$V_3 = L (X^T \hat{\Sigma} X)^{-1} X^T K,\quad V_4 = L (X^T X)^{-1} X^T Y P_{Z^T} Y^T Q_X K$$

$$V_5 = K^T X (X^T \hat{\Sigma} X)^{-1} X^T K,\quad V_6 = L (X^T X)^{-1} X^T Y P_{Z^T} Y^T X (X^T X)^{-1} L^T$$

2 Example

The data of this example is the dental Data[6] that were made on 11 girls and 16 boys at ages 8, 10, 12 and 14 years. Each measurement is the distance from the center of the pituitary to the pteygomaxillary fissure. Since the measurements are obtained at equal time intervals, we can choose the design matrix X taking as the following form:

$$X=\begin{pmatrix}1 & 1 & 1 & 1\\ -3 & -1 & 1 & 3\end{pmatrix}$$

The design matrix Z is then a 1× 27 vector which consists of all 1, i. e. $Z=(1, \cdots, 1)\in R^{27}$.

After calculating, the index plots of some measurements are shown in the following figures. $H_{max}(1)$ is the eigenvector associated with the largest absolute eigenvalue of $D^{(1)}$. The figure 1 indicates point 21 is influential point. $H_{max}(2)$ is the eigenvector associated with the largest absolute eigenvalue of $D^{(4)}$. The figure 2 indicates point 24 is influential point.

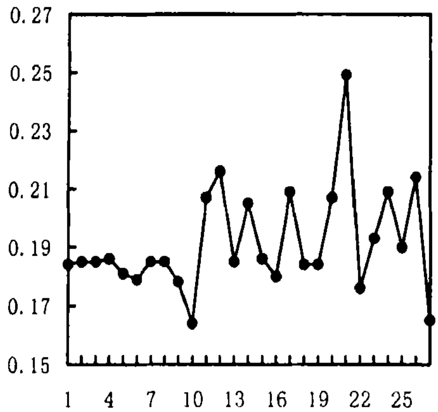

Fig 1 Index plot of Hmax

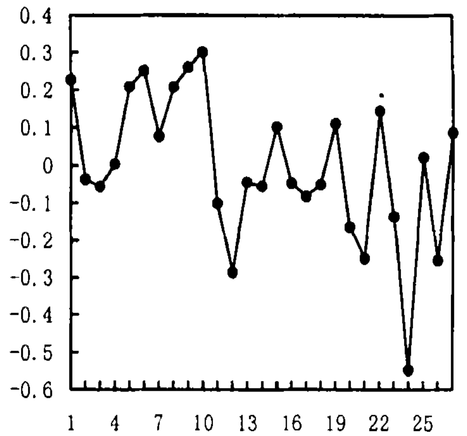

Fig 2 Index plot of Hmax

References:

[1] Rotthoff R F, Roy S N. A generalized multivariate analysis of variance model useful especially for growth curve problems[J]. Biometrika, 1964. 51: 313~ 326.

[2] Kariya T. Testing in the multivariate general linear model[M]. Tokyo: kinokuniya comp, 1985. 203~ 212.

[3] Shi L. local influence in principal components analysis[J]. Biometrika, 1997. 84: 175~ 186.

[4] Dai L, Chen J B, wang X R. local influence of growth curve model with uniform covariance structure[J]. Yunnan Vniversity, 2000. 22(2): 88 ~ 91.

[5] Muirhead R J. Aspects of multivariate statistical theory[M]. New York: John Wiley and Sons, 1986. 47~ 50.

[6] Rao C R. . Prediction of future observations in growth curve model[J]. statistical Science, 1987. 2: 434~ 471.

具有 Rao 的协方差结构的增长曲线模型的局部影响分析

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摘要: 利用关于矩阵的广义影响函数和广义 Cook 统计量来研究协方差阵为 Rao 的简单结构时, 各种扰动形式对增长曲线模型系数矩阵 B 的极大似然估计的局部影响分析, 并用实例说明了该方法的有效性.

关键词: 增长曲线模型, Rao 的简单结构; 广义影响函数; 广义 Cook 统计量.