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奇异半线性反应扩散方程组的 Blow-up 问题

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摘要: 利用算子半群理论和压缩映射原理,对一类奇异半线性反应扩散方程组解的问题进行讨论,得到其解在有限时间内爆破.

关键词: 反应扩散方程组; 算子半群; 比较不等式
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Blow-up Problem of Singular Semilinear Reaction-Diffusion System

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Abstract: The theory of operator and compression mapping principle is adopted in this paper to study the problem of Singular Semi-linear reaction diffusion systems. It is proven that the solution will blow up in finite time.

Key words: reaction-diffusion system; semigroup of operator; comparison inequality

0 引 言

本文主要讨论如下具有奇异系数的半线性反应扩散方程组在有限时间的 Blow-up 问题.

$$\begin{cases} \dfrac{\partial u}{\partial t} - \dfrac{1}{t^\sigma}\Delta u = \alpha_1 u^{q_1} + \beta_1 v^{p_1} + f_1(x) & 0 < \varepsilon_0 < t, x \in R^N \\ \dfrac{\partial v}{\partial t} - \dfrac{1}{t^\sigma}\Delta v = \alpha_2 u^{q_2} + \beta_2 v^{p_2} + f_2(x) & 0 < \varepsilon_0 < t, x \in R^N \\ \lim\limits_{t\to 0^+} u(t,x) = \lim\limits_{t\to 0^+} v(t,x) = 0 \end{cases} \tag{1}$$

其中 $\sigma > 0$ 且 $\sigma \neq 1$, $0 < p_i < 1, q_i > 1$; $\alpha_i > 0, \beta_i > 0$; $f_i(x)$ $(i = 1,2)$ 连续非负有界函数且不恒为零, Δ 是 N 维拉普拉斯算子.

关于奇异半线性反应扩散方程解的问题. 文献[1,2]已经做出了大量研究成果,彭大衡在此基础上研究了 $p_i > 1, q_i > 1$ $(i = 1,2)$ 时的情形,得出了非负局部解存在的几个充分条件及解的爆破结果,在证明解的爆破问题中,他采用的是文献[1,2]证明的方法. 而本文用算子半群理论和压缩映射原理研究了 $0 < p_i < 1, q_i > 1$ $(i = 1,2)$ 时的爆破情况,在原有的方法上有所改进.

1 预备知识与引理

若(1)在带形域 $S_\tau = [0,T) \times R^N$ 存在非负解 $(u(t,x); v(t,x))$ 则

$$\begin{cases} u(t,x) = \varphi_1(t,x) + \int_0^t e^{(\xi(\varepsilon_0) - \xi(s))\Delta}(\alpha_1 u^{q_1} + \beta_1 v^{p_1})\,ds \\ v(t,x) = \varphi_2(t,x) + \int_0^t e^{(\xi(s) - \xi(t))\Delta}(\alpha_2 u^{q_2} + \beta_2 v^{p_2})\,ds \end{cases} \tag{2}$$

其中 $\varphi_i(t,x) = \int_0^t e^{(\xi(s) - \xi(t))\Delta} f_i(x)\,ds$ $(i = 1,2)$, $\xi(s) = \dfrac{s^{1-\sigma}}{\sigma - 1}$ $e^{\tau\Delta}$ 是由拉普拉斯算子 Δ 生成的半群.

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定义 设$f(x)$是连续有界函数,如果对任意给的$\varepsilon > 0(\varepsilon < t)$有$\lim_{\varepsilon\to 0}\int_{\varepsilon}^{t} e^{(\xi(s)-\xi(t))\Delta}f(x)\,\mathrm{d}s$在$L^{\infty}(R^N)$中存在,则积分$\int_0^t e^{(\xi(s)-\xi(t))\Delta}f(x)\,\mathrm{d}s$收敛.并记

$$\int_0^t e^{(\xi(s)-\xi(t))\Delta}f(x)\,\mathrm{d}s = \lim_{\varepsilon\to 0}\int_{\varepsilon}^{t} e^{(\xi(s)-\xi(t))\Delta}f(x)\,\mathrm{d}s$$

假设问题(1)在$S_\tau = [0,T)\times R^N$存在非负解$(w(t,x);z(t,x))$,考虑如下的初值问题

$$\begin{cases}\dfrac{\partial w}{\partial t} - \dfrac{1}{t^{\sigma}}\Delta w = \alpha_1 w^{q_1} + \beta_1 z^{p_1} + f_1(x)\\ \dfrac{\partial z}{\partial t} - \dfrac{1}{t^{\sigma}}\Delta z = \alpha_2 z^{q_2} + \beta_2 w^{p_2} + f_2(x)\\ \lim\limits_{t\to\varepsilon_0} w(t,x) = u(\varepsilon_0,x)\quad \lim\limits_{t\to\varepsilon_0} z(t,x) = v(\varepsilon_0,x)\end{cases} \tag{3}$$

其中$p_i,q_i;\alpha_i,\beta_i;f_i(x)\ (i=1,2)$同问题(1)所设:

$$\left.\begin{aligned} u(\varepsilon_0,x) &= \int_0^{\varepsilon_0} e^{(\xi(\varepsilon_0)-\xi(s))\Delta}(\alpha_1 u^{q_1}(s,x) + \beta_1 v^{p_1}(s,x) + f_1(x))\,\mathrm{d}s\\ v(\varepsilon_0,x) &= \int_0^{\varepsilon_0} e^{(\xi(s)-\xi(t))\Delta}(\alpha_2 u^{q_2}(s,x) + \beta_2 v^{p_2}(s,x) + f_2(x))\,\mathrm{d}s\end{aligned}\right\} \tag{4}$$

引理1[2] 假设$f_i(x)\ (i=1,2)$连续非负有界且不恒为零,$(u(t,x);v(t,x))$为问题(1)式在$S_\tau = [0,T)\times R^N$的非负解;则对任意的$\iota\in(0,T)$,存在$\alpha,\beta>0$使得

$$u(\iota,x)\geqslant \beta e^{-\alpha|x|^2},\ v(\iota,x)\geqslant \beta e^{-\alpha|x|^2} \tag{4}$$

引理2 若$f(x)\in L^{\infty}(R^N)$,则$\left\|\int_0^t e^{(\xi(s)-\xi(t))\Delta}f(x)\,\mathrm{d}s\right\|_{\infty} \leqslant \|f(x)\|_{\infty} t$

引理3 假设问题(3)在$[\varepsilon_0,T)\times R^N$中存在非负解$(u(t,x);v(t,x))$,则存在正的常数$c(p_i,q_i;\alpha_i,\beta_i)\ (i=1,2)$使得

$$(t-\varepsilon_0)^{\frac{1}{(r-1)}}\|e^{(\xi(s)-\xi(t))\Delta}u(t,o)\|_{\infty}\leqslant c(p_i,q_i;\alpha_i,\beta_i) \tag{5}$$

$$(t-\varepsilon_0)^{\frac{1}{(r-1)}}\|e^{(\xi(s)-\xi(t))\Delta}v(t,o)\|_{\infty}\leqslant c(p_i,q_i;\alpha_i,\beta_i) \tag{6}$$

其中$r=\max\{q_1;q_2\}$.

证明 因为$w(t,x),z(t,x)$如(3)所设:

$$w(t,x) = e^{(\xi(\varepsilon_0)-\xi(t))\Delta}u(\varepsilon_0,x) + \int_{\varepsilon_0}^{t} e^{(\xi(t)-\xi(s))\Delta}(\alpha_1 w^{q_1}(s,x) + \beta_1 z^{p_1}(s,x) + f_1(x))\,\mathrm{d}s$$

$$w(t,x)\geqslant e^{(\xi(\varepsilon_0)-\xi(t))\Delta}u(\varepsilon_0,x) \tag{7}$$

$$w(t,x)\geqslant \int_{\varepsilon_0}^{t} e^{(\xi(s)-\xi(t))\Delta}\alpha_1 w^{q_1}(s,x)\,\mathrm{d}s \tag{8}$$

(7)代入(8):

$$w(t,x)\geqslant \alpha_1\int_{\varepsilon_0}^{t} e^{(\xi(s)-\xi(t))\Delta}(e^{(\xi(s)-\xi(t))\Delta}u(\varepsilon_0,x))^{q_1}\,\mathrm{d}s$$

由Jensen不等式有

$$w(t,x)\geqslant \alpha_1\int_{\varepsilon_0}^{t}(e^{(\xi(s)-\xi(t))\Delta}(e^{(\xi(s)-\xi(t))\Delta}u(\varepsilon_0,x)))^{q_1}\,\mathrm{d}s = \alpha_1(t-\varepsilon_0)(e^{(\xi(\varepsilon_0)-\xi(t))\Delta}u(\varepsilon_0,x))^{q_1} \tag{9}$$

(9)代入(8):

$$w(t,x)\geqslant \alpha_1^{q_1+1}\int_{\varepsilon_0}^{t}(s-\varepsilon_0)^{q_1}(e^{(\varepsilon_0)-\xi(t))\Delta}u(\varepsilon_0,x))^{q_1^2}\,\mathrm{d}s = \alpha_1^{(q_1+1)}\frac{(t-\varepsilon_0)^{1+q_1}}{1+q_1}(e^{(\varepsilon_0-\xi(t))\Delta}u(\varepsilon_0,x))^{q_1^2}$$

重复上述过程得:

$$w(t,x)\geqslant \alpha^{1+q_1+\cdots+q_1^{k-1}}\frac{(t-\varepsilon_0)^{1+q_1+\cdots+q_1^{k-1}}(e^{(\xi(\varepsilon_0)-\xi(t))\Delta}u(\varepsilon_0,x))^{q_1^k}}{(1+q_1)q_1^{k-2}(1+q_1+q_1^2)q_1^{k-3}\cdots(1+q_1+\cdots+q_1^{k-1})} \tag{10}$$

在(10)中令$k\to\infty$得:

$$(t-\varepsilon_0)^{1/(q_1-1)}\left(e^{(\xi(\varepsilon_0)-\xi(t))\Delta}u(\varepsilon_0,x)\right)\leqslant \alpha_1^{q_1-1}\prod_{l=2}^{\infty}\left(\frac{q_1^l-1}{q_1-1}\right)^{\frac{1}{q_1^l}} \tag{11}$$

因
$$\ln\prod_{l=2}^{\infty}\left(\frac{q_1^l-1}{q_1-1}\right)^{\frac{1}{q_1^l}}=\sum_{l=2}^{\infty}q_1^{-l}\ln\left(\frac{q_1^l-1}{q_1-1}\right)\leqslant\sum_{l=2}^{\infty}q_1^{-l}\ln(lq_1^l)<\infty$$

在(11)中记 $c(q_1,\alpha_1)=\alpha_1^{q_1-1}\prod_{l=2}^{\infty}\left(\frac{q_1^l-1}{q_1-1}\right)^{\frac{1}{q_1^l}}$，(5)式成立.

同理(6)式也成立.

2 主要结果

定理　假设 $f_i(x)\in L^\infty(R^N)\ (i=1,2)$ 连续有界非负且不恒为零，当 $0<p_i<1<q_i, p_iq_i>1, \frac{N}{2}(q_i-1)\leqslant 1, \frac{q_i+1}{p_iq_i-1}\geqslant\frac{N}{2}$ 时问题(1)式的解必在有限时间内爆破.

证明　不失一般性，假设 $f_1(x), f_2(x)$ 不恒为零. 采用反证法，假设(3)在任何的 $S_\tau=[0,T]\times R^N$ 中存在 $(u(t,x);v(t,x))$；由引理知，任意 $\varepsilon_0\in(0,T)$，存在 $\alpha,\beta>0$ 使得 $u(\varepsilon_0,x)\geqslant\beta e^{-\alpha|x|^2}$，$v(\varepsilon_0,x)\geqslant\beta e^{-\alpha|x|^2}$ 成立，进一步假设知 $(u(t,x);v(t,x))$ 是下述问题在 $[\varepsilon_0,T)\times R^N$ 中的有界正解.

$$\begin{cases}\dfrac{\partial u}{\partial t}-\dfrac{1}{t^\sigma}\Delta u=\alpha_1u^{q_1}+\beta_1v^{p_1}+f_1(x) & 0<\varepsilon_0<t, x\in R^N\\ \dfrac{\partial v}{\partial t}-\dfrac{1}{t^\sigma}\Delta v=\alpha_2u^{q_2}+\beta_2v^{p_2}+f_2(x) & 0<\varepsilon_0<t, x\in R^N\\ \lim\limits_{t\to\varepsilon_0}u(t,x)=u(\varepsilon_0,x)\ \lim\limits_{t\to\varepsilon_0}v(t,x)=v(\varepsilon_0,x)\end{cases}$$

在 $[\varepsilon_0,T)\times R^N$ 中存在有界正解，且

$$e^{(\xi(\varepsilon_0)-\xi(t))\Delta}e^{-\alpha|x|^2}=(1+4\alpha(\xi(\varepsilon_0)-\xi(t)))^{-\frac{N}{2}}\exp\left[-\alpha|x|^2/(1+4\alpha(\xi(\varepsilon_0)-\xi(t)))\right] \tag{12}$$

由(4)式和(12)式得：

$$v(t,x)\geqslant e^{(\xi(\varepsilon_0)-\xi(t))\Delta}v(\varepsilon_0,x)\geqslant\beta(1+4\alpha(\xi(\varepsilon_0)-\xi(t)))^{-\frac{N}{2}}\exp\left[-\alpha|x|^2/(1+4\alpha(\xi(\varepsilon_0)-\xi(t)))\right]$$

$$u(t,x)\geqslant e^{(\xi(\varepsilon_0)-\xi(t))\Delta}v(\varepsilon_0,x)\geqslant\beta(1+4\alpha(\xi(\varepsilon_0)-\xi(t)))^{-\frac{N}{2}}\exp\left[-\alpha|x|^2/(1+4\alpha(\xi(\varepsilon_0)-\xi(t)))\right]$$

又因为

$$\begin{aligned}u(t,x)&\geqslant\int_{\varepsilon_0}^{t}e^{(\xi(s)-\xi(t))\Delta}(\alpha_1u^{q_1}+\beta_1v^{p_1})\,\mathrm{d}s\\&\geqslant\int_{\varepsilon_0}^{t}e^{(\xi(s)-\xi(t))\Delta}\alpha_1\beta^{q_1}(1+4\alpha(\xi(\varepsilon_0)-\xi(t)))^{\frac{-Nq_1}{2}}\exp\left[-\alpha q_1|x|^2/(1+4\alpha(\xi(\varepsilon_0)-\xi(t)))\right]\mathrm{d}s\\&\quad+\int_{\varepsilon_0}^{t}e^{(\xi(s)-\xi(t))\Delta}\beta_1\beta^{p_1}(1+4\alpha(\xi(\varepsilon_0)-\xi(t)))^{\frac{-Np_1}{2}}\exp\left[-\alpha p_1|x|^2/(1+4\alpha(\xi(\varepsilon_0)-\xi(t)))\right]\mathrm{d}s\\&\geqslant\int_{\varepsilon_0}^{t}\alpha_1\beta^{q_1}(1+4\alpha(\xi(\varepsilon_0)-\xi(s)))^{\frac{-N(q_1-1)}{2}}(1+4\alpha(\xi(\varepsilon_0)-\xi(s))+4\alpha q_1(\xi(s))-\xi(t)))^{\frac{-N}{2}}\\&\quad\exp\left[-\alpha q_1|x|^2/(1+4\alpha(\xi(\varepsilon_0)-\xi(s))+4\alpha q_1(\xi(s)-\xi(t))))\right]\mathrm{d}s+\int_{\varepsilon_0}^{t}\beta_1\beta^{p_1}(1+\\&\quad4\alpha(\xi(\varepsilon_0)-\xi(s)))^{\frac{-N(p_1-1)}{2}}(1+4\alpha(\xi(\varepsilon_0)-\xi(s))+4\alpha p_1(\xi(s)-\xi(t)))^{\frac{-N}{2}}\\&\quad\exp\left[-\alpha q_1|x|^2/(1+4\alpha(\xi(\varepsilon_0)-\xi(s))+4\alpha q_1(\xi(s)-\xi(t))))\right]\mathrm{d}s\end{aligned} \tag{13}$$

令 $h(s)=(1+4\alpha(\xi(\varepsilon_0)-\xi(s))+4\alpha q_1(\xi(s)-\xi(t)))\quad(\varepsilon_0\leqslant s\leqslant t)$

$h'(s)=-4\alpha\xi'(s)(1-q_1)=\frac{4\alpha}{s^\sigma}(1-q_1)<0$，$h(s)$ 单调递减，

$m(s)=(1+4\alpha(\xi(\varepsilon_0)-\xi(s))+4\alpha p_1(\xi(s)-\xi(t)))\quad(\varepsilon_0\leqslant s\leqslant t)$ 单调递增，

即
$$\begin{aligned}1+4\alpha(\xi(\varepsilon_0)-\xi(t))&\leqslant 1+4\alpha(\xi(\varepsilon_0)-\xi(s))+4\alpha q_1(\xi(s)-\xi(t)\\&\leqslant 1+4\alpha q_1(\xi(\varepsilon_0)-\xi(t))\end{aligned}$$

$$1+4\alpha q_1(\xi(\varepsilon_0)-\xi(t))\leqslant(1+4\alpha(\xi(s)-\xi(t)))+4\alpha q_1(\xi(s)-\xi(t))\leqslant(1+4\alpha(\xi(\varepsilon_0)-\xi(t)))$$

$$u(t,x)\geqslant\alpha_1\beta^{q_1}\int_{\varepsilon_0}^{t}(1+4\alpha(\xi(\varepsilon_0)-\xi(s)))^{\frac{-N(q_1-1)}{2}}(1+(4\alpha q_1(\xi(\varepsilon_0)-\xi(t)))^{\frac{-N}{2}}$$
$$\exp[-\alpha q_1|x|^2/(1+4\alpha q_1(\xi(\varepsilon_0)-\xi(t))]) \mathrm{d}s+\beta_1\beta^{p_1}\int_{\varepsilon_0}^{l}(1+4\alpha(\xi(\varepsilon_0)-\xi(s))^{\frac{-N(p_1-1)}{2}}$$
$$(1+4\alpha(\xi(\varepsilon_0)-\xi(t))^{\frac{-N}{2}}\exp[-\alpha p_1|x|^2/(1+4\alpha(\xi(\varepsilon_0)-\xi(t))))]\mathrm{d}s$$

因为 $\frac{N}{2}(q_1-1)\leqslant 0$，$-\frac{N}{2}(p_1-1)>0$，

$$u(t,x)\geqslant\alpha_1\beta^{q_1}(1+(4\alpha q_1(\xi(\varepsilon_0)-\xi(t)))^{\frac{-N}{2}}\exp[-\alpha q_1|x|^2/(1+4\alpha q_1(\xi(\varepsilon_0)-\xi(t))^{q_1})]$$
$$\int_{\varepsilon_0}^{t}(1+4\alpha(\xi(\varepsilon_0)-\xi(s))^{-1}\mathrm{d}s+\beta_1\beta^{p_1}\int_{\varepsilon_0}^{l}(1+4\alpha(\xi(\varepsilon_0)-\xi(t)))^{\frac{-N}{2}}$$
$$\exp[-\alpha p_1|x|^2/(1+4\alpha(\xi(\varepsilon_0)-\xi(t))))]\int_{\varepsilon_0}^{l}(1+4\alpha(\xi(\varepsilon_0)-\xi(s)))^{\frac{-N(p-1)}{2}}\mathrm{d}s$$

$\xi(s)=\frac{s^{1-\sigma}}{\sigma-1}\quad(s>0)$， $\xi'(s)=-\frac{1}{S^{\sigma}}<0$， $\xi(s)$ **单调递减**

$\xi(\varepsilon_0)\geqslant\xi(s)$， $\xi(\varepsilon_0)-\xi(s)\geqslant 0$，

$$u(t,x)\geqslant\alpha_1\beta^{q_1}(1+(4\alpha q_1(\xi(\varepsilon_0)-\xi(t)))^{\frac{-N}{2}}\exp[-\alpha q_1|x|^2/(1+4\alpha q_1(\xi(\varepsilon_0)-\xi(t))))]$$
$$\int_{0}^{l}(1+4\alpha(\xi(S)-\xi(s)))^{-1}\mathrm{d}s+\beta_1\beta^{p_1}(1+4\alpha(\xi(\varepsilon_0)-\xi(t)))^{\frac{-N}{2}}$$
$$\exp[-\alpha p_1|x|^2/(1+4\alpha(\xi(\varepsilon_0)-\xi(t))))]\int_{\varepsilon_0}^{l}(1+4\alpha(\xi(\varepsilon_0)-\xi(\varepsilon_0)))^{\frac{-N(p_1-1)}{2}}\mathrm{d}s\geqslant$$
$$\alpha_1\beta^{q_1}(1+(4\alpha q_1(\xi(\varepsilon_0)-\xi(t)))^{\frac{-N}{2}}\exp[-\alpha q_1|x|^2/(1+4\alpha q_1(\xi(\varepsilon_0)-\xi(t)))](t-\varepsilon_0)+$$
$$\beta_1\beta^{p_1}(1+(4\alpha(\xi(\varepsilon_0)-\xi(t)))^{\frac{-N}{2}}\exp[-\alpha p_1|x|^2/(1+4\alpha(\xi(\varepsilon_0)-\xi(t)))](t-\varepsilon_0)\geqslant$$
$$A(t-\varepsilon_0)\exp(-\alpha q_1|x|^2)+B(t-\varepsilon_0)\exp(-\alpha p_1|x|^2).$$

同理 $v(s,t)\geqslant \mathrm{C}(t-\varepsilon_0)\exp(-\alpha q_2|x|^2)+\mathrm{D}(t-\varepsilon_0)\exp(-\alpha p_2|x|^2)$

$$(t-\varepsilon_0)^{1/(r-1)}\|e^{(\xi(s)-\xi(t))\Delta}u(t,x)\|_\infty\geqslant A(t-\varepsilon_0)^{1/(r-1)+1}\|e^{(\xi(s)-\xi(t))\Delta}\exp(-\alpha q_1|x|^2)\|_\infty+$$
$$B(t-\varepsilon_0)^{1/(r-1)+1}\|e^{(\xi(s)-\xi(t))\Delta}\exp(-\alpha p_2|x|^2)\|_\infty$$

$$(t-\varepsilon_0)^{1/(r-1)+1}\|e^{(\xi(s)-\xi(t))\Delta}u(t,0)\|_\infty\geqslant A(t-\varepsilon_0)^{1/(r-1)+1}\|e^{(\xi(s)-\xi(t))\Delta}1\|_\infty+B(t-\varepsilon_0)^{1/(r-1)+1}\|e^{(\xi(s)-\xi(t))\Delta}1\|_\infty$$
$$\geqslant(A+B)(t-\varepsilon_0)^N=M(t-\varepsilon_0)^N \quad (16)$$

同理 $(t-\varepsilon_0)^{1/(r-1)+1}\|e^{(\xi(s)-\xi(t))\Delta}v(t,0)\|_\infty\geqslant(C+D)(t-\varepsilon_0)^N=N(t-\varepsilon_0)^N$ (17)

取适当的 t，**取任意的** $c(p_i,q_i;\alpha_i,\beta_i)$ **可使得** $\min\{M(t-\varepsilon_0)^N,N(t-\varepsilon_0)^N\}>c(p_i,q_i;\alpha_i,\beta_i)$ **所以**(16)(17)**分别与**(5)(6)**矛盾;问题**(1)**的非负解必在有限时间内爆破**.

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