Adjustment and Accuracy Estimation for GPS Control Networks without Strong Base Datum

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Abstract: It is not ignorable that there is base datum with less precision to support the network computation and adjustment of a constrained GPS network. With the help of modern geodesy techniques GPS and adjustment theory as well as computational techniques, a new method is put forward to carry out the adjustment of GPS network with less precision to support the computation. Based on the theory and model, a small GPS network is computed in the southwest of China to test the theory and model. Based upon the comparison, suggestions are also made in this paper.

Key words: GPS network adjustment; GPS technique; adjustment model
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起始数据有误差的 GPS控制网的平差与精度估算

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摘要: 在处理如 GPS控制网的约束平差或以 GPS测定导线网的方位角的平差计算中,是否考虑起始数据的误差给平差结果及其精度评定带来的影响是不可忽视的. 通过平差模型的推导及数值计算的结果对 GPS网中起始数据有误差的平差与精度估算这一问题进行了讨论,提出了一种新方法来实施 GPS网的平差,并给出了实施此类计算的建议.

关键词: GPS网平差; GPS技术;平差模型

0 Introduction

According to the case really occurred that most cities horizontal controlling network has been established with traditional way or advanced GPS techniques originally in China, and in order to conform to the standard issued by the Chinese Construction Department in 1997, The Technical Regulations for the Urban Surveying with GPS. It is reasonable to adapt the original datum to be the reference frame when we carry out the surveying engineering and designing surveying network with GPS techniques, and the original datum and existed large scale maps must be considered. How to make full use of the reasonable parts of the original reference system in a local city and develop the high accuracy information of data observed with GPS economically is the problem this paper deals with. And with time going, this problem will be more and more important in China.

1 Adjustment model

The observations of GPS are composed of space vectors that are surveyed with different time intervals. They

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can be expressed as $(\Delta x, \Delta y, \Delta z)$, or differential coordinates of two arbitrary points on the ground. However, we may meet the case that there is base datum with not enough precision to support the network computation and adjustment of a constrained GPS network.

The ways to get the solution of base datum with not enough precision can be approached by following

1. 1 Strict adjustment model for the base datum with not enough precision

Adjustment strictly according to the principle of least square. i. e. adjustment base datum with not enough precision combined to the GPS observations. It is not a convenient and economic method to be use in real work.

1. 2 Available approximation adjustment model

Supposing the precision of the base datum is enough in adjustment of GPS network, estimating the contribution of the base datum with not enough precision when evaluation of the computation the final results. It will be shown this is a better and economic way in constrained adjustment of GPS network.

1. 2. 1 Strict adjustment model for the base datum with not enough precision

In following discussion we take it as acknowledge that base datum with not enough precision is a kind of observation with a prior - weight, and it is adjusted with general GPS vector observation. According to the principle of least square, we can write the indirect adjustment model as following

$$V_l = A\delta Y + C\delta X - l$$
$$V_s = \delta \hat{X} + L_s \quad (1)$$

and

$$l = AY^0 + CX^0 + l$$
$$L_s = 0 \quad (2)$$

In above formula, $\delta\hat{Y}$ are the parameter vectors of GPS network, $\delta\hat{X}$ are the base datum parameter vector. And

$$P = \begin{bmatrix} P_l & 0 \\ 0 & P_s \end{bmatrix} \quad (3)$$

Suppose

$$V = \begin{bmatrix} V_l \\ V_s \end{bmatrix}, \quad B = \begin{bmatrix} A & C \\ 0 & E \end{bmatrix}, \quad \hat{Z} = \begin{bmatrix} \delta\hat{Y} \\ \delta\hat{X} \end{bmatrix}, \quad l = \begin{bmatrix} l \\ l_s \end{bmatrix}.$$

Formula (1) will be

$$V = B\hat{Z} + l \quad (4)$$

We can then compose the normal equation based upon (4) as follows,

$$B^T PB\hat{Z} + B^T Pl = 0 \quad (5)$$

Suppose

$$N = B^T PB, \quad W = B^T Pl$$

We have

$$N = \begin{bmatrix} A^T & 0 \\ C^T & E \end{bmatrix} \begin{bmatrix} P_L & 0 \\ 0 & P_x \end{bmatrix} \begin{bmatrix} A & C \\ 0 & E \end{bmatrix} = \begin{bmatrix} A^T P_L A & A^T P_L C \\ C^T PA & C^T P_L C + P_x \end{bmatrix}$$

$$W = \begin{bmatrix} A^T P_L l \\ C^T P_L l \end{bmatrix}$$

Then we have

$$\hat{Z} = -N^{-1}W \quad (6)$$

and

$$Q_{ZZ} = N^{-1}$$

suppose,

$$N = \begin{bmatrix} A^T P_L A & A^T P_L C \\ C^T PA & C^T P_L C + P_x \end{bmatrix} = \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \quad (7)$$

and,

$$Q_{ZZ} = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix}$$

then

$$NQ_{ZZ} = E = \begin{bmatrix} E_1 & 0 \\ 0 & E_2 \end{bmatrix}$$

By the law of reversal matrix, we can obtain the result as follows

$$Q_{11} = N_{11}^{-1} + N_{11}^{-1} N_{12} N_{22}^{-1} N_{21} N_{11}^{-1}$$

$$Q_{12} = -N_{11}^{-1} N_{12} N_{22}^{-1} Q_{21}^{T}$$

$$Q_{22} = N_{22}^{-1} = (N_{22} - N_{21} N_{11}^{-1} N_{12})^{-1}$$

Suppose there is a function of parameters as following,

$$\hat{\phi} = \hat{\phi}(\hat{Z}), \quad d\hat{\phi} = f_Y^T \delta \hat{Y} + f_Y^T \delta \hat{X} \tag{8}$$

The covariance will be

$$Q_{\phi\phi} = [f_Y^T \quad f_X^T] \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \begin{bmatrix} f_Y \\ f_X \end{bmatrix} \tag{9}$$

1. 2. 2 Available approximation adjustment model

Supposing the precision of the base datum is enough in adjustment of GPS network, estimating the contribution of the base datum with not enough precision when evaluation of the computation the final results. According to the principle of indirective adjustment model, we can have

$$V = A\delta Y - l, \ p_l \tag{10}$$

and

$$A^T p_l A \delta \hat{Y} + A^T p_l l = 0$$

$$N\delta \hat{Y} + W = 0 \tag{11}$$

we have

$$\delta \hat{Y} = -N^{-1} W \tag{12}$$

$$Q_{\hat{Y}\hat{Y}} = N^{-1}$$

Suppose there is a function of parameters as following

$$\hat{\phi} = \hat{\phi}(\hat{y}), \quad d\hat{\phi} = f^T \delta \hat{y} + f_\lambda^T d\lambda \tag{13}$$

The covariance will be

$$Q_{\hat{\phi}\hat{\phi}} = f^t q + \phi_\lambda^T Q_{\lambda\lambda} \phi_l \tag{14}$$

and

$$\phi = p_l A_q$$

$$\phi = -(A^T P_l A\lambda)^T q + f_\lambda = -A_\lambda^T \phi + f_\lambda$$

and

$$A^T P_l A_q - f = 0$$

2 Example

A small GPS network in southwest of China in which there are two base datum points with not enough precision. And their covariance matrix is $\begin{bmatrix} 0.6 & 0.2 \\ 0.2 & 0.4 \end{bmatrix}$, there are four independent observations l_1, l_2, l_3, l_4 and their covariance are 2, 1, 2, 2. We are going to make adjustment with available approximation adjustment model and strict adjustment model for the base datum with not enough precision.

2. 1 Strict adjustment model

By formula (1), we have

$$v=\begin{bmatrix}1&0\\-1&1\\0&1\\0&-1\end{bmatrix}\begin{bmatrix}\delta\hat{X}_2\\\delta\hat{X}_3\end{bmatrix}+\begin{bmatrix}-1&0\\0&0\\0&0\\0&-1\end{bmatrix}\begin{bmatrix}\delta\hat{X}_1\\\delta\hat{X}_4\end{bmatrix}+l,\quad v_x=\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}\delta\hat{X}_1\\\delta\hat{X}_4\end{bmatrix}$$

We can obtain

$$p_x=Q_X^{-1}=\begin{bmatrix}2&-1\\-1&3\end{bmatrix},\ p_l=Q_L^{-1}=\begin{bmatrix}0.5&0&0&0\\0&1&0&0\\0&0&0.5&0\\0&0&0&0.5\end{bmatrix}$$

$$Q_{11}=\begin{bmatrix}0.9757&0.4796\\0.4796&0.7320\end{bmatrix},\ Q_{12}=\begin{bmatrix}0.0322&-0.0008\\0.0270&-0.0001\end{bmatrix},\ Q_{22}=\begin{bmatrix}0.0460&0.0093\\0.0093&0.0312\end{bmatrix}$$

$$d\hat{\varphi}=[0\quad 6]\begin{bmatrix}\delta\hat{X}_2\\\delta\hat{X}_3\end{bmatrix}+[0\quad -1]\begin{bmatrix}\delta\hat{X}_1\\\delta\hat{X}_4\end{bmatrix}$$

$$Q_{\hat{\varphi}\hat{\varphi}}=f_y^tQ_{11}f_Y+f_y^tQ_{21}f_Y+f_Y^TQ_{12}f_Y$$

$$Q_{\hat{\varphi}\hat{\varphi}}=0.7320+0.0001+0.0001+0.0312=0.7634$$

2.2 Available approximation adjustment model

By means of formula (9), we have

$$V=\begin{bmatrix}1&0\\-1&1\\0&1\\0&-1\end{bmatrix}\begin{bmatrix}\delta\hat{X}_2\\\delta\hat{X}_3\end{bmatrix}+l,\ A_\lambda=\begin{bmatrix}-1&0\\0&0\\-1&0\\0&1\end{bmatrix}$$

$$\begin{bmatrix}1.5&-1\\-1&2\end{bmatrix}\begin{bmatrix}q_1\\q_2\end{bmatrix}-\begin{bmatrix}0\\1\end{bmatrix}=0$$

$$\begin{bmatrix}q_1\\q_2\end{bmatrix}=\begin{bmatrix}1.5&-1\\-1&1.5\end{bmatrix}^{-1}\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}0.50\\0.75\end{bmatrix}$$

$$\varphi_\lambda=-A_\lambda^TP_LA_q+f_\lambda=\begin{bmatrix}0.625\\-0.625\end{bmatrix}$$

$$Q_{\varphi\varphi}=f^tq+\varphi_\lambda^TQ_{\lambda\lambda}\varphi_\lambda=[0\quad 1]\begin{bmatrix}0.50\\0.75\end{bmatrix}+[0.625\quad -0.625]$$

$$\begin{bmatrix}0.6&0.2\\0.2&0.4\end{bmatrix}\begin{bmatrix}0.625\\-0.625\end{bmatrix}=0.75+0.23=0.92$$

3 Summary and suggestions

1) Compare the two $Q_{\varphi\varphi}$ from the two ways, available approximation adjustment model is better than strict adjustment model for the base datum with not enough precision. And the most important thing is that the former does not change the base datum. But the later will change the base datum and this will destroy the uniform of the original local frame;

2) The quantity of computation for the available approximation adjustment model is much less and simple than strict adjustment model;

3) The unit standard deviation (usd) in the two models is different. We should pay enough attention to this fact. They are

$$\sigma_{01}^2=\frac{V^TP_LV+V_X^TP_XV_X}{n-t+t_l}$$

and

$$\sigma_{02}^2=V^TP_LV/(n-t)$$

4) The matching of prior - weight must be considered before adjustment in two models for estimation of precision. And we suggest surveying engineer apply the available approximation adjustment model in real work.

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